

North Gauhati College
Department of Mathematics

SEMESTER III(HONOURS)
ASSIGNMENT I

MAT-HC-3016
Theory of Real Functions

August 2021

TOTAL MARKS: 25

INSTRUCTIONS TO CANDIDATES

1. This assignment paper contains **Seven (7)** questions and comprises **Two (2)** printed pages.
2. Each question carry **Five** marks. Answer any **Five** of all questions.
3. Submit the assignment as a single **PDF** file through the online portal of our college website under section “Assignments” and send a copy to the email id mathngc1969@gmail.com.
4. Write your **Name**, **GU Roll No.**, and **Registration Number** in the assignment .
5. Submission **Due Date** is on or before **10th August, 2021**.

(Answer any **Five**)

1. Show that a number $c \in \mathbb{R}$ is a cluster point of a subset A of \mathbb{R} if and only if there exists a sequence (a_n) in A such that $\lim(a_n) = c$ and $a_n \neq c$ for all $n \in \mathbb{N}$.

2. Let $I = [a, b]$ be a closed bounded interval and let $f : I \rightarrow \mathbb{R}$ be continuous on I . Then f is bounded on I .

3. Prove that a function f defined on an interval I is continuous at $a \in I$ if and only if for every sequence (a_n) in I which converges to a , we have $\lim_{n \rightarrow \infty} f(a_n) = f(a)$.

4. Define uniform continuity of a function on an interval. Prove that every uniformly continuous function on an interval is continuous on that interval. Justify with an example that the converse is not true.

5. Show that the function f defined by

$$f(x) = \begin{cases} -x & \text{if } x \text{ is rational} \\ x & \text{if } x \text{ is irrational} \end{cases}$$

is continuous only at $x = 0$.

6. Prove that $f(x) = \sin(x^2)$ is not uniformly continuous on $[0, \infty)$.

7. Show that

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$$

is derivable at $x = 0$ but

$$\lim_{x \rightarrow 0} f'(x) \neq f'(0).$$

END OF PAPER