

North Gauhati College
Sub: Mathematics
Semester: 4 (Major)
Paper: 4016(Multivariate Calculus)
Marks: 30

1. (a) Define continuity of a function of two variable on a set S . 2
- (b) Test the continuity of the function f at $(0, 0)$, where
 - i. $f(x, y) = \frac{x-y}{x^2+y^2}$ 1
 - ii. $f(x) = \begin{cases} y \sin \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ 3
- (c) Compute the partial derivative $\frac{\partial^2 f}{\partial x \partial y}$ of the function $f(x, y) = 5x^2 - 2xy + 3y^2$. 2
2. (a) Define the terms relative maximum, relative minimum, absolute maximum and absolute minimum. 2
- (b) Find all relative extrema and saddle points of the function $f(x, y) = 2x^2 + 2xy + y^2 - 2x - 2y + 5$. 5
3. (a) Compute the area of the region D bounded by the line $y = x$ and below by the circle $x^2 + y^2 - 2y = 0$. 4
- (b) Find the surface area of the portion of the plane $x + y + z = 1$ that lies in the first octant (where $x \geq 0, y \geq 0, z \geq 0$). 4
4. (a) State Green's theorem. 2
- (b) Show that Green's theorem is true for the line intgral $\oint_C (-ydx + xdy)$, where C is the closed upper half semi-circle with centre at $(0, 0)$ and radius 1. 5