North Gauhati College

Department of Mathematics

Sessional Examination 2021

Real Analysis (MAT-HG-4016/ MAT-RC-4016)

QUIZ #1

Full Marks: 30 Time Duration: $1\frac{1}{2}$ hour

INSTRUCTIONS TO CANDIDATES

1. This question paper contains **Seven (7)** questions and comprises **Two (2)** printed pages.
2. Answer all the questions.
3. Write your **Name**, **GU Roll No.**, and **Registration Number** .
4. Submit the solutions as a single **PDF** file through the online portal of our college website under section "**Assignments**".

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1. Answer the following as directed. [1×4]
   (a) State true or false: The sequence $((-1)^n)$ is convergent.
   (b) Write the value of the limit:
   $$\lim \left(1 + \frac{1}{n}\right)^n$$
   .
   (c) State Monotone Subsequence Theorem.
   (d) Write the definition of Cauchy sequence.
2. Prove that limit of a sequence is unique. [3]
3. Show that $\lim(\frac{1}{n^2+1}) = 0$. [3]

4. Suppose that $X = (x_n)$, $Y = (y_n)$, and $Z = (z_n)$ are sequences of real numbers such that

$$x_n \leq y_n \leq z_n \quad \text{for all } \; n \in \mathbb{N},$$

and that $\lim(x_n) = \lim(z_n)$. Then prove that $Y = (y_n)$ is convergent and

$$\lim(x_n) = \lim(y_n) = \lim(z_n).$$

Further show that

$$\lim \left(\frac{\sin n}{n} \right) = 0.$$

[3+2]

5. State and prove Monotone Convergence Theorem. [5]

6. Let $Z = (z_n)$ be the sequence of real numbers defined by

$$z_1 = 1, \; z_{n+1} = \sqrt{2z_n}$$

for $n \in \mathbb{N}$. Show that $\lim(z_n) = 2$. [5]

7. Prove that a bounded sequence of real numbers has a convergent subsequence. [5]

END OF PAPER