North Gauhati College
Department of Mathematics

SEMESTER III(HONOURS)
HOME ASSIGNMENT

MAT-HC-3016
Theory of Real Functions

January 2022 TOTAL MARKS: 30

INSTRUCTIONS TO CANDIDATES

1. This assignment paper contains **Eight (8)** questions and comprises **Two (2)** printed pages.
2. Each question carry **Five** marks. Answer any **Six** of all questions.
3. Submit the assignment as a single **PDF** file through the online portal of our college website under section "Assignments" and submit a hard copy in the Department of Mathematics.
4. Write your **Name**, **GU Roll No.**, and **Registration Number** in the assignment .
5. Submission **Due Date** is on or before 22nd January, 2022.

(*Answer any* ***Six***)

1. Show that a number $c \in \mathbb{R}$ is a cluster point of a subset A of $\mathbb{R}$ if and only if there exists a sequence (a_n) in A such that $\lim(a_n) = c$ and $a_n \neq c$ for all $n \in \mathbb{N}$.

2. Let $I = [a, b]$ be a closed bounded interval and let $f : I \to \mathbb{R}$ be continuous on I. Then prove that f is bounded on I.

3. Prove that a function f defined on an interval I is continuous at $a \in I$ if and only if for every sequence (a_n) in I which converges to a, we have $\lim_{n\to\infty} f(a_n) = f(a)$.

4. Define uniform continuity of a function on an interval. Prove that every uniformly continuous function on an interval is continuous on that interval. Justify with an example that the converse is not true.

5. Show that the function f defined by

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

is not continuous at any point of $\mathbb{R}$.

6. Let I be a closed bounded interval and let $f : I \to \mathbb{R}$ be continuous on I. Then prove that f is uniformly continuous on I.

7. State and prove Bolzano's Intermediate Value Theorem.

8. Let $I = [a, b]$ be a closed bounded interval and let $f : I \to \mathbb{R}$ be continuous on I. Then prove that f has an absolute maximum and an absolute minimum on I.

END OF PAPER