North Gauhati College
Department of Mathematics

SEMESTER V(HONOURS)
HOME ASSIGNMENT 2022

MAT-HC-5026
Linear Algebra

January 2022 TOTAL MARKS: 30

INSTRUCTIONS TO CANDIDATES

1. This assignment paper contains **Eight (8)** questions and comprises **Two (2)** printed pages.
2. Each question carry **Five** marks. Answer any **Six** of all questions.
3. Submit the assignment as a single **PDF** file through the online portal of our college website under section "Assignments" and submit a hard copy in the Department of Mathematics.
4. Write your **Name**, **GU Roll No.**, and **Registration Number** in the assignment .
5. Submission **Due Date** is on or before 22nd January, 2022.

(*Answer any* ***Six***)

1. If a vector space V has a basis $B = \{b_1, \cdots, b_n\}$, then prove that any set in V containing more than n vectors must be linearly dependent.

2. Find the dimension of the subspace

$$H = \left\{ \begin{bmatrix} a - 3b + c \\ 5a + 4d \\ b - 2c - d \\ 5d \end{bmatrix} : a, b, c, d \in \mathbb{R} \right\}$$

3. Let V be a p-dimensional vector space, $p \geq 1$. Prove that any linearly independent set of exactly p elements in V is automatically a basis for V. Further, any set of exactly p elements that spans V is automatically a basis for V.

4. Let $B = \{b_1, \cdots, b_n\}$ be a basis for a vector space V. Then prove that the coordinate mapping $x \to [x]_B$ is a one-to-one linear transformation from V onto $\mathbb{R}^n$.

5. Find a spanning set for the null space of the matrix

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$$

6. Let $S = \{v_1, \cdots, v_p\}$ be a set in V, and let $H = Span\,\{v_1, \cdots, v_p\}$. Then prove the following:

a. If one of the vectors in S—say, v_k is a linear combination of the remaining vectors in S, then the set formed from S by removing v_k still spans H.

b. If $H \neq \{0\}$, some subset of S is a basis for H.

7. Prove that the pivot columns of a matrix A form a basis for $ColA$.

8. Let $b_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $b_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, $x = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$, and $B = \{b_1, b_2\}$. Find the coordinate vector $[x]_B$ of x relative to B.

END OF PAPER