North Gauhati College

Department of Mathematics

Semester I (Honours)

Home Assignment 2022

MAT-HC-1026

Algebra

January 2022 Total Marks: 30

INSTRUCTIONS TO CANDIDATES

1. This assignment paper contains **two(2)** questions and comprises of **two(2)** printed pages.
2. Mark against each question is indicated at right hand side of concerned question.
3. Submit the assignment as a single **PDF** file through the online portal of our college website under section "Assignments" and submit a hard copy in the Department of Mathematics.
4. Write your **Name** and **Class Roll No.** in the assignment.
5. Submission **Due Date** is on or before **30th January, 2022**.

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1. Answer the following questions : 5×4=20
   (a) Define contrapositive statement. Using it, prove that "For an integer n, if $n^3 - 1$ is even, then n is odd."
   (b) For given sets A and B, prove that the following are equivalent:
      i. $A \subseteq B$
      ii. $A \cap B = A$
      iii. $A \cup B = B$
      iv. $B^c \subseteq A^c$
   (c) Let $f : X \to Y$ and $g : Y \to Z$ be two functions. Prove the following:
      i. If f and g are injective, then $g \circ f$ is injective.
      ii. If f and g are surjective, the $g \circ f$ is surjective.

(d) Let $f : [0,1] \to [0,1]$ be a function defined by $f(x) = \frac{1-x}{1+x}$. Show that f is a bijection. Also find its inverse.

2. State Well Ordering Principle. Let $a, b \in \mathbb{Z}$ with $a \in \mathbb{N}$. Then prove that there exists unique integers q and r such that

   (a) $b = aq + r$, and

   (b) $0 \leq r < a$.

2+8=10