North Gauhati College

Department of Mathematics

Semester V (Honours)

Home Assignment 2022

MAT-HC-5016

Riemann Integration and Metric Spaces

January 2022 Total Marks: 30

INSTRUCTIONS TO CANDIDATES

1. This assignment paper contains **six(6)** questions and comprises of **two(2)** printed pages.
2. Mark against each question is indicated at right hand side of concerned question.
3. Submit the assignment as a single **PDF** file through the online portal of our college website under section "Assignments" and submit a hard copy in the Department of Mathematics.
4. Write your **Name**, **Class Roll No.**, **GU Roll No.** and **Registration No.** in the assignment.
5. Submission **Due Date** is on or before **31th January, 2022**.

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1. Let $f(x) = x$ on $[0, 1]$. Let $P = \{x_i = \frac{i}{4} : i = 0, 1, 2, 3, 4\}$ and $Q = \{x_j = \frac{j}{8} : j = 0, 1, 2, ..., 8\}$. Find $L(f, P)$, $L(f, Q)$, $U(f, P)$ and $U(f, Q)$. [4]
2. Prove that a bounded function $f : [a, b] \to \mathbb{R}$ is Darboux integrable if and only if for each $\epsilon > 0$, there exists a partition P of $[a, b]$ such that [5]

$$U(f, P) - L(f, P) < \epsilon.$$

3. Prove the following: [5]
   (a) A continuous function $f : [a, b] \to \mathbb{R}$ is Darboux integrable.
   (b) A bounded and monotone function $f : [a, b] \to \mathbb{R}$ is Darboux integrable.

4. Define Cauchy sequence in a metric space. Prove that a convergent sequence in a metric space is a Cauchy sequence. Give an example to show that the converse is not true. [5]

5. Define complete metric space. Let $X = C[a, b]$ be the space of continuous functions on $[a, b]$ and $d(f, g) = sup\{|f(x) - g(x)| : a \leq x \leq b\}$ be the associated metric. Show that (X, d) is a complete metric space. [5]

6. State and prove Baire Category Theorem. [6]

End of assignment paper