North Gauhati College
Department of Mathematics

Semester V(DSE 1)
Home Assignment 2022

MAT-HE-5016
Number Theory

January 2022   **Total Marks: 30**

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1. Answer the following questions: $[4\times1=4]$

   (a) Define number theoretic function.

   (b) Find $\tau(n)$ and $\rho(n)$ when $n = 360$.

   (c) $[-\pi] = ?$, where $[n]$ denotes the greatest integer less than or equal to n.

   (d) Calculate $\phi(360)$.

2. Prove that the functions τ and ρ are both multiplicative functions. $[5]$

3. State and prove the Möbius Inversion Formula. [5]

4. Define Euler's ϕ-function. For any integer $n > 1$, if n has the prime factorization $n = p_1^{k_1} p_2^{k_2} ... p_r^{k_r}$, then show that [5]

$$\phi(n) = n(1 - \frac{1}{p_1})(1 - \frac{1}{p_2})...(1 - \frac{1}{p_r}).$$

5. If n is a positive integer and p is a prime, the show that the exponent of the highest power of p that divides $n!$ is [5]

$$\sum_{k=1}^{\infty} [n/p^k].$$

6. Prove the following statements: [6]

   (a) If $n > 2$, then $\phi(n)$ is an even integer.

   (b) If n is an odd integer, then $\phi(2n) = \phi(n)$.

End of assignment paper