

North Gauhati College
Department of Mathematics

Semester V(DSE 1)
Home Assignment 2022

MAT-HE-5016
Number Theory

January 2022

Total Marks: 30

INSTRUCTIONS TO CANDIDATES

1. This assignment paper contains **six(6)** questions and comprises of **two(2)** printed pages.
 2. Mark against each question is indicated at right hand side of concerned question.
 3. Submit the assignment as a single **PDF** file through the online portal of our college website under section "Assignments" and submit a hard copy in the Department of Mathematics.
 4. Write your **Name, Class Roll No., GU Roll No. and Registration No.** in the assignment.
 5. Submission **Due Date** is on or before **31th January, 2022**.
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1. Answer the following questions: [4×1=4]
 - (a) Define number theoretic function.
 - (b) Find $\tau(n)$ and $\rho(n)$ when $n = 360$.
 - (c) $[-\pi] = ?$, where $[n]$ denotes the greatest integer less than or equal to n .
 - (d) Calculate $\phi(360)$.
2. Prove that the functions τ and ρ are both multiplicative functions. [5]

3. State and prove the Möbius Inversion Formula. [5]

4. Define Euler's ϕ -function. For any integer $n > 1$, if n has the prime factorization $n = p_1^{k_1} p_2^{k_2} \dots p_r^{k_r}$, then show that [5]

$$\phi(n) = n \left(1 - \frac{1}{p_1}\right) \left(1 - \frac{1}{p_2}\right) \dots \left(1 - \frac{1}{p_r}\right).$$

5. If n is a positive integer and p is a prime, the show that the exponent of the highest power of p that divides $n!$ is [5]

$$\sum_{k=1}^{\infty} [n/p^k].$$

6. Prove the following statements: [6]

(a) If $n > 2$, then $\phi(n)$ is an even integer.

(b) If n is an odd integer, then $\phi(2n) = \phi(n)$.

End of assignment paper